EXAMEN 2^oA MAT. APLIC. II

SOLUCIÓN Salvo error u omisión 11-5-10

1°) $$s(t) = \frac{340 + 330t - 10t^2}{t+2} \qquad t \neq -2$$

a/ Igualo a cero

$$s(t) = 0 \Rightarrow 340 + 330t - 10t^2 = 0 \Rightarrow t^2 - 33t - 34 = 0$$

$$\Rightarrow t = \frac{33 \pm \sqrt{33^2 + 4 \cdot 34}}{2} = \frac{33 \pm \sqrt{1225}}{2} = \frac{33 \pm 35}{2} = \begin{cases} 34 \\ -1 \end{cases}$$

el valor pedido es $t = 34$

b/ Derivo

$$s'(t) = \frac{(330 - 20t)(t+2) - (340 + 330t - 10t^2)}{(t+2)^2} =$$

$$= \frac{330t + 660 - 20t^2 - 40t - 340 - 330t + 10t^2}{(t+2)^2} = \frac{-10t^2 - 40t + 320}{(t+2)^2}$$

Veo ptos críticos

$$s'(t) = 0 \Rightarrow -10t^2 - 40t + 320 = 0 \Rightarrow t^2 + 4t - 32 = 0 \Rightarrow$$

$$\Rightarrow t = \frac{-4 \pm \sqrt{16 + 4 \cdot 32}}{2} = \frac{-4 \pm \sqrt{144}}{2} = \frac{-4 \pm 12}{2} = \begin{cases} 4 \\ -8 \end{cases}$$

Veo signo de $s'(t)$

	-8		-2		4	
$-$		$+$		$+$		$-$
$+$		$+$		$+$		$+$
$-$		$+$		$+$		$-$
$\searrow$		$\nearrow$		$\nearrow$		$\searrow$

El máximo se alcanza en $t = 4$

c/ A. Verticales $t = -2$

$$\lim_{t \to -2^-} s(t) = \frac{-}{-}\infty = +\infty \qquad \lim_{t \to -2^+} s(t) = \frac{-}{+}\infty = -\infty$$

A Horizontales no hay

A Oblicuas $y = mt + b$

$$m = \lim_{t \to \pm\infty} \frac{s(t)}{t} = \lim_{t \to \pm\infty} \frac{340 + 330t - 10t^2}{t^2 + 2t} = -10$$

$$b = \lim_{t\to\pm\infty} S(t) - mt = \lim_{t\to\pm\infty}\left(\frac{340+330t-10t^2}{t+2} + 10t\right) =$$

$$= \lim_{t\to\pm\infty} \frac{340+350t}{t+2} = 350$$

$$\Rightarrow \quad y = -10t + 350$$

2º a/ $$\lim_{x\to 4} \frac{x^2-2x-8}{-\frac{x^2}{2}+x+4} = \lim_{x\to 4} \frac{2x^2-4x-16}{-x^2+2x+8} \to \frac{0}{0}$$

factorizo

$$2x^2-4x-16=0 \qquad\qquad -x^2+2x+8=0$$

$$x^2-2x-8=0 \qquad\qquad x^2-2x-8=0$$

$$x = \frac{2\pm\sqrt{4+32}}{2} \begin{matrix} \nearrow 4 \\ \searrow -2 \end{matrix}$$

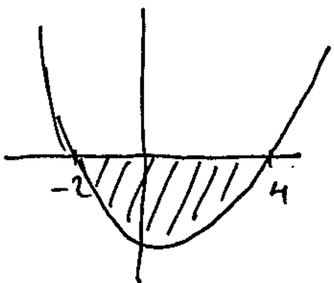
$$\lim_{x\to 4} \frac{2(x-4)(x+2)}{-(x+2)(x-4)} = -2$$

b/ El área limitada entre $f(x)$ y $g(x)$ es el área limitada entre $f(x)-g(x)$ y el eje OX

$$f(x)-g(x) = x^2-2x-8-\left(\frac{-x^2}{2}+x+4\right) = \frac{3x^2}{2}-3x-12$$

Represento viendo ptos de corte con eje OX

$$\frac{3x^2}{2}-3x-12=0 \Rightarrow x^2-2x-8=0 \Rightarrow x = \begin{matrix} \nearrow 4 \\ \searrow -2 \end{matrix}$$



Calculo

$$I = \int_{-2}^{4} \frac{3x^2}{2}-3x-12\,dx =$$

$$= \frac{x^3}{2} - \frac{3x^2}{2} - 12x \Big|_{-2}^{4} = \frac{4^3}{2} - \frac{3\cdot 4^2}{2} - 12\cdot 4 - \left(\frac{(-2)^3}{2} - \frac{3(-2)^2}{2} - 12(-2)\right) =$$

$$= 32-24-48-(-4-6+24) = -54$$

$$\Rightarrow A = 54\,u^2$$

③ a/ la bisectriz del primer cuadrante es la recta $y=x$. Tiene pendiente 1.

Busco un punto de la curva con derivada 1

$\Rightarrow y' = 2x = 1 \Rightarrow x = \frac{1}{2} \Rightarrow y = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

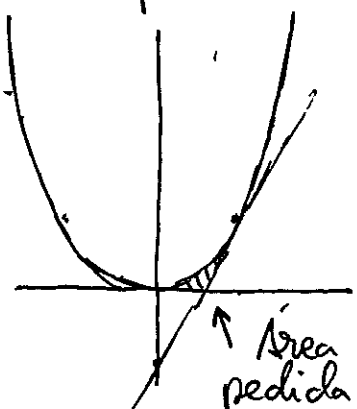
las coordenadas son $\left(\frac{1}{2}, \frac{1}{4}\right)$

b/ Calculo la tangente a la curva en $(1,1)$

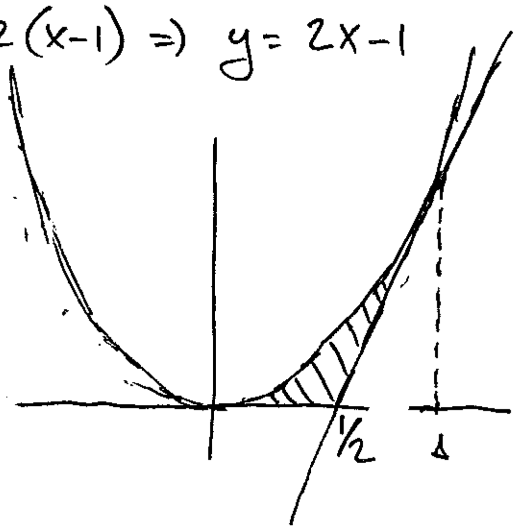
$m =$ y' en $x=1 \Rightarrow m = 2$

la recta es $y-1 = 2(x-1) \Rightarrow y = 2x-1$

Represento.



Amplío un poquito



Calculo el área bajo la parábola y le resto el área del triángulo bajo la recta

$$A = \int_0^1 x^2\,dx - \frac{b \cdot h}{2} = \frac{x^3}{3}\Big|_0^1 - \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

④

$$y' = \frac{1}{Ln 10} \cdot \frac{1}{\frac{x^4-3x}{\sqrt{x}}} \cdot \frac{(4x^3-3)\cdot\sqrt{x} - (x^4-3x)\cdot\frac{1}{2\sqrt{x}}}{x}$$

$$y' = 2^{\frac{sen^2 x}{x}} \cdot Ln 2 \cdot \frac{2\, sen x \cdot cos x \cdot x - sen^2 x}{x^2}$$

(5°) a/ $\int \frac{x-1}{x^2-2x+3}\,dx = \frac{1}{2}\int \frac{2(x-1)}{x^2-2x+3}\,dx =$

$= \frac{1}{2}\int \frac{2x-2}{x^2-2x+3} = \frac{1}{2}\cdot \ln|x^2-2x+3| + K$

b/ $\int \frac{x^2+x-\sqrt{x}}{x}\,dx = \int \frac{x^2}{x} + \frac{x}{x} - \frac{\sqrt{x}}{x}\,dx =$

$= \int x - 1 - x^{-1/2}\,dx = \frac{x^2}{2} - x - \frac{x^{1/2}}{1/2} + K =$

$= \frac{x^2}{2} - x - 2\sqrt{x} + K$

(6°) $f(x) = \frac{x^2}{4-x^2}$ Dom $f(x) = \mathbb{R} - \{\pm 2\}$

a/ A. Verticales $x=-2$, $x=2$

Veo límites laterales

$\lim_{x\to -2^-} \frac{x^2}{(2+x)(2-x)} = \frac{+}{-\cdot+}\infty = -\infty$ $\lim_{x\to 2^-} f(x) = \frac{+}{++}\infty = +\infty$

$\lim_{x\to -2^+} f(x) = \frac{+}{++}\infty = +\infty$ $\lim_{x\to 2^+} f(x) = \frac{+}{+\cdot-}\infty = -\infty$

A. Horizontales

$\lim_{x\to\pm\infty} \frac{x^2}{4-x^2} = -1 \Rightarrow y = -1$

No hay oblicuas

b/ Derivo

$f'(x) = \frac{2x(4-x^2) - x^2(-2x)}{(4-x^2)^2} = \frac{8x - 2x^3 + 2x^3}{(4-x^2)^2} =$

$= \frac{8x}{(4-x^2)^2}$

Veo puntos críticos

$f'(x) = 0 \Rightarrow 8x = 0 \Rightarrow x = 0$

Estudio signo de $f'(x)$

$-$		$-$	0	$+$		$+$
$+$		$+$		$+$		$+$
$-$	-2	$-$		$+$	2	$+$
$\searrow$	-2	$\searrow$	0	$\nearrow$	2	$\nearrow$

Hay un mínimo en $x = 0 \Rightarrow y = 0$

Crece $\forall x \in (0,2) \cup (2,\infty)$

Decrece $\forall x \in (-\infty,-2) \cup (-2,0)$

c/